# KLT TRACKING USING INTRINSIC AND EXTRINSIC CAMERA PARAMETERS IN CONSIDERATION OF UNCERTAINTY

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Abstract: Feature tracking is an important task in computer vision, especially for 3D reconstruction applications. Such procedures can be run in environments with a controlled sensor, e.g. a robot arm with camera. This yields the camera parameters as special knowledge that should be used during all steps of the application to improve the results. As a first step, KLT (Kanade-Lucas-Tomasi) tracking (and its variants) is an approach widely accepted and used to track image point features. So, it is straightforward to adapt KLT tracking in a way that camera parameters are used to improve the feature tracking results. The contribution of this work is an explicit formulation of the KLT tracking procedure incorporating known camera parameters. Since practical applications do not run without noise, the uncertainty of the camera parameters is regarded and modeled within the procedure. Comparing practical experiments have been performed and the results are presented.

### **1 INTRODUCTION**

### 1.1 Problem Statement and Motivation

The 3D reconstruction of objects from digital images is a still unsolved problem, that has an important role for many industrial applications. Especially hardware systems containing a sensor mounted on a controlled element (robot arm or equivalent), yielding positional sensor parameters, are widely used (cf. (Kuehmstedt et al., 2001)). Using this kind of set-up, it is shown (Wenhardt et al., 2006) that the reconstruction result can be improved, if the reconstruction process is embedded in a next best view planning approach. But without active illumination, all these reconstruction methods suffer from the correspondence problem, i.e. the identification of image points mapped from one 3D world point.

For a pair of stereo images and known camera (intrinsic and extrinsic) parameters, stereo matching may be performed by scanning the other image's corresponding horizontal line for one point within the rectified image pair. But the above mentioned applications for 3D reconstruction provide video streams by nature. Thus, feature point tracking is the way most commonly used to collect image point correspondences (like in (Wenhardt et al., 2006)) within the image sequence. These feature point tracking methods, like KLT tracking, have been developed with respect to the structure-from-motion approach. Therefore, they ignore camera parameters.

All feature point tracking methods aim to find the mappings of *one 3D world point* into several images. Without any knowledge of the camera poses or without using that knowledge, tracking algorithms are bound to work appearance-based only. KLT tracking is doing so by minimizing the sum of squared errors between the pixel intensity values of two *patches* (small image regions). There is no reference to the corresponding 3D world point at all, and hence, the well-known motion drift problem (Rav-Acha and Peleg, 2006) can occur. In addition, a lot of care has to be taken for the selection of good features to track (Shi and Tomasi, 1994).

Addressing the mentioned problems is the contribution of this paper. This is done by explicitly incorporating knowledge about the camera (intrinsic and extrinsic parameters) into the parameterization and optimization process of KLT tracking. The search space for patches in consecutive frames is restricted by the epipolar constraint. Hence, the above mentioned ways to establish point correspondences are merged in order to create a new solution to the correspondence problem for 3D reconstruction with a controlled sensor.

The remainder of the paper is organized as follows. In section 2 the parameterization and optimization process of KLT tracking is described. The incorporation of the epipolar constraint (by using intrinsic and extrinsic camera parameters as prior knowledge) is demonstrated in section 3. Section 4 shows, how the uncertainty of the epipolar geometry is given attention to and modeled within the extended tracker. Experimental results are demonstrated in section 5, and the paper is concluded in the last section.

#### **1.2 Literature Review**

The original idea of tracking features by an iterative optimization process was presented by Lucas and Kanade in (Lucas and Kanade, 1981). Since then a rich variety of adaptations and extension has been published, giving rise to surveys like (Baker and Matthews, 2004). (Fusiello et al., 1999) deal with the removal of spurious corespondences by using robust statistics. The problem of reselection of the template image is dealt with in (Zinsser et al., 2005).

Since these modifications and extensions are independent from applying camera parameters, only very few of them are mentioned. For more information the reader may be referred to (Baker and Matthews, 2004).

## 2 KLT TRACKING

In this section the basic equations of KLT tracking are derived and summarized as far as needed for the remainder of the paper. This can also be found in (Baker and Matthews, 2004).

Under the assumptions of constant image brightness (see (Cox et al., 1995)) and a small baseline between consecutive frames, the pixel-wise sum of squared intensity differences between small image regions (patches)  $T(\mathbf{x})$  from the first image and  $I(\mathbf{x})$  from the second image defines an error  $\varepsilon$ . The functions  $T(\mathbf{x})$  and  $I(\mathbf{x})$  yield the intensity values at pixel position  $\mathbf{x} = (x, y)^T$  in the respective image region P. Now, the error  $\varepsilon$  is parameterized by a vector  $\mathbf{p}$ . The entries of this vector are used for the defined geometrical warping  $W(\mathbf{x}, \mathbf{p})$  from  $T(\mathbf{x})$  to  $I(W(\mathbf{x}, \mathbf{p}))$ . Thus, the error is

$$\varepsilon(\mathbf{p}) = \sum_{\mathbf{x} \in P} (I(W(\mathbf{x}, \mathbf{p})) - T(\mathbf{x}))^2.$$
(1)

The warping function  $W(\mathbf{x}, \mathbf{p})$  may perform different geometrical transformations. Common choices are pure translation (thus,  $\mathbf{p} = (p_1, p_2)^T$  containing two parameters for translation within the image plane, namely in image x- and y-direction), affine transformation (six parameters) or projective transformation (eight parameters).

Within the iterative optimization process, where an initial allocation of **p** is already known, equation (1) is reparameterized with  $\Delta \mathbf{p}$  to

$$\varepsilon(\Delta \mathbf{p}) = \sum_{\mathbf{x} \in P} (I(W(\mathbf{x}, \mathbf{p} + \Delta \mathbf{p})) - T(\mathbf{x}))^2, \quad (2)$$

also known as compositional approach. In order to solve for  $\Delta \mathbf{p}$ , two first-order Taylor approximations are performed, yielding (for details the reader is referred to (Baker and Matthews, 2004))

$$\varepsilon'(\Delta \mathbf{p}) = \sum_{\mathbf{x} \in P} (I(W(\mathbf{x}, \mathbf{p})) + \nabla I \frac{\partial W(\mathbf{x}, \mathbf{p})}{\partial \mathbf{p}} \Delta \mathbf{p} - T(\mathbf{x}))^2,$$
(3)

where  $\frac{\partial W(\mathbf{x},\mathbf{p})}{\partial \mathbf{p}}$  is the Jacobian of  $W(\mathbf{x},\mathbf{p})$ , with  $\varepsilon(\Delta \mathbf{p}) \approx \varepsilon'(\Delta \mathbf{p})$ . For the purpose of minimization, the first derivative of equation (3) is set to zero. Hence, the optimization rule is

$$\Delta \mathbf{p} = H^{-1} \sum_{\mathbf{x} \in P} \left( \nabla I \frac{\partial W(\mathbf{x}, \mathbf{p})}{\partial \mathbf{p}} \right)^T \left( T(\mathbf{x}) - I(W(\mathbf{x}, \mathbf{p})) \right)$$
(4)

with the Hessian

$$H = \sum_{\mathbf{x} \in P} \left( \nabla I \frac{\partial W(\mathbf{x}, \mathbf{p})}{\partial \mathbf{p}} \right)^T \left( \nabla I \frac{\partial W(\mathbf{x}, \mathbf{p})}{\partial \mathbf{p}} \right).$$
(5)

By equation (4) an optimization rule is defined for computing  $\mathbf{p}_{i+1}$  from  $\mathbf{p}_i$ , namely  $\mathbf{p}_{i+1} = \mathbf{p}_i + \Delta \mathbf{p}$ .

## 3 USING INTRINSIC AND EXTRINSIC CAMERA PARAMETERS

In this section the reparameterization of the warping function  $W(\mathbf{x}, \mathbf{p})$  by using camera parameters (intrinsic and extrinsic) as prior knowledge is described. The additional knowledge is used to compute the epipolar geometry (cf. (Hartley and Zisserman, 2003)) of consecutive frames. Then the translational part of the warping function is modified so that the template patch can only be moved along the corresponding epipolar line. With respect to clarity and w.l.o.g. the warping function is assumed to perform a pure translation, since the modifications do not affect the affine or projective part of the transformation. The treatment of affine and projective parameters remains the same as for the standard KLT tracker.

For the computation of the fundamental matrix *F* from camera parameters the reader is referred to (Hartley and Zisserman, 2003). Once calculated, the position of a point **x** in the first image can be restricted to the corresponding epipolar line  $\mathbf{l} = (l_1, l_2, l_3)^T$  in the second image. The epipolar line  $\mathbf{l}$  is given by  $\mathbf{l} = F\tilde{\mathbf{x}}$  with  $\tilde{\mathbf{x}} = (x, y, 1)^T$ . A parameterized form of this line is

$$\mathbf{l}(\lambda) = \begin{pmatrix} \frac{-l_3}{l_1} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -l_2 \\ l_1 \end{pmatrix}$$
(6)

with parameter  $\lambda$ . Thus, for pure translation the new epipolar warping function is given by

$$W_E(\mathbf{x}, \mathbf{p}) = \begin{pmatrix} \frac{-l_3}{l_1} - \lambda l_2 \\ \lambda l_1 \end{pmatrix}, \tag{7}$$

using  $\mathbf{l} = F\tilde{\mathbf{x}}$  and  $\mathbf{p} = \lambda$ . In the case of  $l_1$  being close to zero, another parameterization of  $\mathbf{l}$  has to be used. Equation (7) shows the reparameterization of the translational transformation regarding the epipolar constraint. The Jacobian of this expression is simply

$$\frac{\partial W_E(\mathbf{x}, \mathbf{p})}{\partial \mathbf{p}} = \begin{pmatrix} \frac{\partial W_{E,x}(\mathbf{x}, \mathbf{p})}{\partial \lambda} \\ \frac{\partial W_{E,y}(\mathbf{x}, \mathbf{p})}{\partial \lambda} \end{pmatrix} = \begin{pmatrix} -l_2 \\ l_1 \end{pmatrix}.$$
 (8)

Using equation (8) in the optimization rule from equations (4) and (5), the adaptation to the case of known camera parameters is reached. For the moment, the translation of a pixel between two frames is strictly limited to the movement along the corresponding epipolar line (expressed by parameter  $\lambda$ ), reducing the optimization search space by one degree of freedom.

## 4 IN CONSIDERATION OF UNCERTAINTY

Up to now, the warping function for one pixel is only allowing for movements on the corresponding epipolar line. But with respect to noisy camera parameters and to discretization, a possible deviation from the epipolar line has to be modeled. This section shows a way to incorporate uncertainty into the parameterization and into the optimization process from equation (4). For the mentioned, obvious reasons the restriction of moving only along the epipolar line has to be softened. This can be achieved by allowing movement perpendicular to the epipolar line. But, with these two linearly independent directions, the search space again covers the whole image plane, which seems to neutralize any advantages reached by the reduction of the number of parameters. Consequently, some mechanism to control the single translational parts (perpendicular to / along the epipolar line) has to be added. This is achieved by a weighting factor  $w \in [0, 1]$ , called *epipolar weight*, controlling the amounts of accepted parameter changes.

With respect to uncertainty the modified epipolar warping function is

$$W_{EU}(\mathbf{x},\mathbf{p}) = \begin{pmatrix} \frac{-l_3}{l_1} - \lambda_1 l_2 + \lambda_2 l_1\\ \lambda_1 l_1 + \lambda_2 l_2 \end{pmatrix}, \quad (9)$$

with  $\mathbf{l} = F \tilde{\mathbf{x}}, \mathbf{p} = (\lambda_1, \lambda_2)^T$  and the Jacobian

$$\frac{\partial W_{EU}(\mathbf{x}, \mathbf{p})}{\partial \mathbf{p}} = \begin{pmatrix} \frac{\partial W_{EU,x}(\mathbf{x}, \mathbf{p})}{\partial \lambda_1} & \frac{\partial W_{EU,x}(\mathbf{x}, \mathbf{p})}{\partial \lambda_2} \\ \frac{\partial W_{EU,y}(\mathbf{x}, \mathbf{p})}{\partial \lambda_1} & \frac{\partial W_{EU,y}(\mathbf{x}, \mathbf{p})}{\partial \lambda_2} \end{pmatrix}$$
$$= \begin{pmatrix} -l_2 & l_1 \\ l_1 & l_2 \end{pmatrix}.$$
(10)

Applying this to the rule from equations (4) and (5), nearly the original optimization is performed, but with the exception of translating along and perpendicular to the corresponding epipolar line and not in image x- and y-direction (for the general case). The epipolar constraint respecting uncertainty is achieved by adding to the optimization rule a weighting matrix

$$A_w = \left(\begin{array}{cc} w & 0\\ 0 & 1-w \end{array}\right) \tag{11}$$

that controls the amount (within each dimension) of the calculated  $\Delta \mathbf{p}$  that is accepted, finally. The modified optimization rule is

$$\Delta \mathbf{p}_{EU,w} = A_w H_{EU}^{-1} S_{EU} \tag{12}$$

with  $H_{EU}$  given by expression (5) with the substitution from equation (10) and

$$S_{EU} = \sum_{\mathbf{x} \in P} \left( \nabla I \frac{\partial W_{EU}(\mathbf{x}, \mathbf{p})}{\partial \mathbf{p}} \right)^T (T(\mathbf{x}) - I(W_{EU}(\mathbf{x}, \mathbf{p}))).$$
(13)

By this specification, the change of translational parameters is optimized with respect to the epipolar geometry. Changes along the epipolar line are accepted with weight w (perpendicular with weight

1 - w) within each optimization step. For the hypothetical case of a perfectly accurate epipolar geometry, w = 1 could be used, resulting in the optimization rule described in section 3. The automatic computation of *w* has not been explored, yet. There might be a way to yield *w* with respect to the uncertainty of the epipolar line calculated from noisy camera parameters.

### **5 EXPERIMENTAL RESULTS**

This section shows experimental results. The standard KLT tracker is compared to the modified tracker described in this work in terms of tracking accuracy and mean trail length of tracked points in an image sequence. As warping function both trackers use the respective variants of pure translation (x-/y-direction,  $\lambda_1$ -/ $\lambda_2$ -direction). The performance of the modified tracker is tested with respect to the epipolar weight *w*.

### 5.1 Trail Length Evaluation

For this experiment an image sequence has been recorded. The calibrated camera was mounted on the hand of a Staeubli RX90L robot arm providing the extrinsic parameters. The image sequence consisted of 21 frames, one for the initialization of the tracker and 20 for tracking. The figures 1 to 3 show some of the 100 features selected (pictures are cut and enlarged for visibility reasons) and two tracking steps. The images are taken from the test run with w = 0.9 set.



Figure 1: Initial frame with 100 image features selected.

The figures show partially different positions of the tracked features. This effect is quantified in the next subsection.



Figure 2: Frame 9. Tracked points by standard KLT marked by light green crosses. Yellow diamonds indicate points of the modified tracker (w = 0.9).



Figure 3: Frame 20.

For each feature point the trail length (number of frames in which the point could be tracked) was stored. From these values the mean trail length and the variance for all points were computed. The results are shown in tables 1 and 2.

Table 1: Mean trail lengths and variances with respect to w. Values for standard tracker: mean 16.07 frames (fr), variance 27.83 frames<sup>2</sup>.

epipolar weight w	0.5	0.6	0.7
mean trail length (fr)	15.96	16.16	16.18
variance (fr <sup>2</sup> )	28.12	26.97	27.11

Table 2: Continuing table 1.

epipolar weight w	0.8	0.9	0.95
mean trail length (fr)	16.10	16.00	16.04
variance (fr <sup>2</sup> )	26.99	27.74	27.64

The values from tables 1 and 2 show comparable performance for the aspect of mean trail length. For

w = 0.7 the mean trail length produced by the modified tracker is about one percent longer then by the standard KLT tracker.

### 5.2 Accuracy Evaluation

Especially with respect to 3D reconstruction, another important characteristic of a feature tracker is the accuracy of the tracked feature points. To compare the accuracy of the modified tracker to the standard KLT tracker, ground truth information has been generated for an image pair (figures 4 and 5). The ground truth correspondences in the second image were blindly (without knowledge about the tracking results) handmarked. Extrinsic camera parameters were calculated by the method proposed in (Trummer et al., 2006).



Figure 4: First frame with 100 features selected.



Figure 5: Second frame. Again, tracked points by standard KLT marked by light green crosses. Yellow diamonds indicate points of the modified tracker (w = 0.5).

Especially along edges the results of the trackers differ from each other. The tracking accuracy is expressed in terms of the mean error distance of a tracked point from its ground truth correspondence. The variance is also given. Tables 3 and 4 show the results for different values of *w*.

Table 3: Mean error distance with respect to w. Values for standard tracker: mean 5.84 pixels (px), variance 51.40 pixels<sup>2</sup>.

epipolar weight w	0.5	0.6	0.7
mean error distance (px)	4.78	4.69	4.97
variance (px <sup>2</sup> )	30.52	32.19	39.80

Table 4: Continuing table 3.

epipolar weight w	0.8	0.9	0.95
mean error distance (px)	4.89	5.37	5.39
variance (px <sup>2</sup> )	48.14	52.32	55.98

With the modified tracker, for each allocation of w the mean error distance is up to one pixel smaller than for the standard KLT tracker. An interesting point is the error value for w = 0.5. In that case, the modified optimization in principal does the same as the standard one. Only the optimization step size is half as wide (w = 0.5) and the translation is optimized along directions  $\lambda_1$  and  $\lambda_2$  (along/perpendicular to the respective epipolar line). But, already this reparameterization of the translation directions has positive influence on the tracking accuracy. The large variances are due to point features along edges, where larger errors may occur. But also this negative effect of the the well-known aperture problem is constricted, if w is chosen properly. With feature points being tracked more accurately, the input data for 3D reconstruction and, thus, the reconstruction result will benefit.



Figure 6: Close-up from figure 4 showing initial features along edge.



Figure 7: Tracking results as close-up from figure 5. Standard tracker (points marked by light green crosses) lost one point, some of the others are drifted along the edge. Modified tracker (yellow diamonds) found all points and preserved point alignment.

The figures 6 and 7 demonstrate more clearly the differences between the results of the compared trackers. By incorporating the epipolar constraint with regard to uncertainty, the modified tracker was able to find one more point in the illustrated region and to keep a better alignment of the tracked feature points. The mean error distance was up to 20 percent smaller (for w = 0.6) using the modified tracker.

## 6 CONCLUSION AND OUTLOOK

In this paper we showed a method to modify the wellknown KLT tracker incorporating knowledge about the extrinsic and intrinsic camera parameters. The additional prior knowledge is utilized to reparameterize the warping function. With respect to noise in practical applications, uncertainty is modeled within the optimization rule. While the mean trail length could only be improved very slightly, the experiments performed show a better accuracy when using the modified tracker. Remarkable is the fact that the epipolar optimization directions alone have a positive effect on the tracking result.

For the future, this modification of the KLT tracker offers lots of further topics to be investigated. Setting the weighting factor w to a certain value may be replaced by an automatic detection concerning the amount of uncertainty of the camera parameters. We also think about changing w during the optimization process.

Another step is the concurrent improvement of accuracy and trail length. At the current stage, accuracy is addressed already. When aiming at longer trail lengths, a closer look at the reasons of losing a feature has to be taken. One of these reasons, surely, is a too large error measured (cf. expression (1)) between corresponding patches. That means, the selected transformation is not able to model all changes between the patches within the error bound set. But with regard to the (soft) epipolar constraint of the modified tracker, this error bound may be raised without the optimization process losing its way. Another possibility to be explored is random jumping along the epipolar line, when a feature is lost.

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