Coarse Registration of 3D Surface Triangulations Based on Moment Invariants with Applications to Object Alignment and Identification

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Abstract

We present a new, direct way to register three-dimensional (3D) surfaces given the respective 3D points and surface triangulations. Our method is non-iterative and does not require any initial solution. The idea is to compute 3D invariants based on local surface moments. The resulting local surface descriptors are invariant with respect to Euclidean or to similarity transformations, by choice. In the final step we use the Hungarian method to find a minimum cost assignment of the computed descriptors. The method is robust against different point densities, noise and partial overlap. Our experiments with real data also show that the method can serve as automatic initialization of the iterative-closest-point (ICP) algorithm and, hence, extends the field of applications for this standard registration method.

1. Introduction

Solving the registration problem for 3D objects is a crucial task in many 3D applications. One important issue in this context is estimating optimal transformations between sets of 3D points, i. e. aligning the two sets. The alignment is necessary, for instance, to fuse partial reconstructions of one object or to compute an error measure with respect to a ground-truth model in another coordinate frame. As another example, the developing research with photonic mixer devices (PMD), like in [9], requires registration of range data. In the example of aligning two partial reconstructions the fact of partial overlap of the input point sets is an additional challenge to the registration method. Considering the alignment of a measurement and the ground-truth model, the registration method used has to cope with different point densities and distributions. As the standard solution to the 3D registration problem, the ICP algorithm [2] produces results depending strongly on a proper initialization [25, 16] and is not explicitly designed to handle input sets with only partial overlap.

We contribute to the solution of the 3D registration problem by computing exact 3D surface moments, which allows the calculation of algebraically derived moment invariants, and by an optimal minimum cost assignment of these invariants. We use local surface moments and propose a method to filter out non-distinctive moments. The method's robustness against noise and the ability to handle partial overlap endow coarse registration as suitable initial solution to the ICP algorithm. Aside from the 3D points, we use the respective 3D surface triangulations that are obtained by standard Delaunay triangulation based on [4]. We perform an optimal point assignment using the Hungarian method [10]. In this way, we extend an existing point matching method presented in [20] to work in 3D space with different surface triangulations of the same object in presence of only partial overlap. A further possible application is object identification. Given a database of 3D objects, we align a candidate object to each object in the database. We yield the identification by comparison of the respective alignment errors.

The remainder of this paper is organized as follows. In the following section we give a review of the relevant literature. We present our registration method in Sect. 3 and evaluate results in Sect. 4. Finally, we conclude the paper by a summary and outlook to future work in Sect. 5.

2. Literature Review

Deriving algebraic moment invariants is originally the work of Hu [8]. In the following years, the set of known moment invariants was increased by invariants based on moments up to arbitrary orders [5]. Providing invariant features of points, lines and other shapes and objects, moment invariants find broad application to solve classification, identification and matching tasks [15]. For a survey of the development and the application of moment invariants see [6]. Voss and Suesse [20] use invariants of 2D point moments to perform matching of affinely transformed 2D point sets. They establish affine invariants based on non-centralized point moments, since the moments and invariants are calculated for each point after setting the origin into the respective point. By this means and an optimal point assignment using the Hungarian method [10] they present a way to perform matching of a 2D point set and an affine transformation of that same point set. Lo and Don [11] algebraically derive a set of 3D moment invariants. The application of 3D moment invariants with parametric surfaces is shown by Xu and Li in [24], but they sample the surface instead of calculating the exact surface moments. While moments are extensively used for 2D and 3D classification and recognition tasks [6], the application of moment invariants to the field of 3D surface registration is only little explored.

The ICP algorithm is established by Besl and McKay [2]. Lots of the extensions to this now standard 3D registration method are compared in [16]. The application of moment invariants from 3D points to the ICP algorithm is shown in [18]. Compared to our work, in [18] the usage of moment invariants is incorporated directly into the iterative estimation procedure of ICP, which still needs an initial solution.

Finding suitable ways of computing a coarse registration of 3D data sets is an actual research topic [23], formerly reviewed in [1]. Some methods aim at assigning features from salient points or surface regions, *e.g.* [12, 22, 17], whereat the features are constructed from geometrical [12, 22] or information-theoretic [17] considerations. Other methods concentrate on effective strategies for searching the whole 3D input data, *e.g.* [3, 23, 21], applying iterative RANSAC matching schemes [3, 21] or by using a volumetric data representation [23]. The registration method in [14] only applies to range data. Compared to these methods, we use a non-iterative scheme to optimally assign invariant features that we calculate directly from the 3D surface without local fitting or matching. We compute exact surface moments allowing the application of algebraically derived moment invariants [11] as invariant features.

3. Registration of 3D Surface Triangulations

In the following we derive a new, direct method to register general 3D surface triangulations that are related at least partially by an unknown Euclidean or similarity transformation.

The main idea is based on the work of Voss and Suesse [20]. They compute non-centralized 2D point moment invariants as descriptors for each point with the origin of the coordinate frame situated in the respective point. Afterwards, an optimal point assignment is established by the Hungarian method using the Euclidean distance between descriptors as the assignment cost. By this means they create a solution to the task of 2D affine point pattern matching.

Our contribution is to extend the matching method [20] from 2D to 3D data to solve the 3D registration problem. For this broad application, we have to face further difficulties:

- Different 3D point clouds, *e.g.* different 3D reconstructions of the same object are in general not the same point sets. Hence the preconditions of any matching algorithm assuming transformations of *one* point set are violated. The 3D point sets may feature different point densities and point distributions. To cope with these differences, we further extend the method [20] to incorporate surface information given as surface triangulations. Unlike Xu and Li [24] we compute exact surface moments without sampling.
- Another challenge is real partial overlap of the two surfaces. We face this problem by calculating moments of a *local* surface around the point considered. Such a local surface region may lack structural information, which rises the *aperture problem*. We handle this problem by a statistical analysis described in Sect. 3.3.

As the concluding step we also use the Hungarian method to assign points based on the descriptors from our local 3D surface moment invariants.

The surface triangulations input to our registration method need not be part of the reconstruction result. So the registration can be applied to any 2.5D or 3D data, at least after an intermediate step for surface triangulation using, for instance, implementations of Delaunay triangulation [4] from `Matlab` or `Meshlab`.

3.1. Computing Moments of 3D Surface Triangulations and 3D Moment Invariants

We consider a 3D surface triangulation $\mathcal{S}$ consisting of triangles $\mathcal{T}_i$, $i \in \mathcal{I} \subset \mathbb{N}$, that are defined by three corner points $\mathbf{c}_1^i, \mathbf{c}_2^i, \mathbf{c}_3^i \in \mathbb{R}^3$ with $\mathbf{c}_j^i = (x_j^i, y_j^i, z_j^i)^T$, $i \in \mathcal{I}$ and $j \in \{1,2,3\}$. The $(k+l+m)^{\text{th}}$ order 3D surface moments M_{klm} of $\mathcal{S}$ are the accumulated surface moments m_{klm}^i of the associated triangles $\mathcal{T}_i$, i. e.

$$M_{klm} = \sum_{i:i \in \mathcal{I}} m_{klm}^i. \tag{1}$$

For a general triangle $\mathcal{T}$ the surface moments are

$$m_{klm} = \iint\limits_{\mathcal{T}} x^k y^l z^m \rho(x,y,z) \mathrm{d}s \tag{2}$$

with a surface density function ρ. We use $\rho(x,y,z) \equiv 1$. With a suitable parameterization $P_{\mathcal{T}}(u,v) = (x_{\mathcal{T}}(u,v), y_{\mathcal{T}}(u,v), z_{\mathcal{T}}(u,v))^T$, $u, v \in D \subset \mathbb{R}^2$, of triangle $\mathcal{T}$, the triangle's surface moments can be written as

$$m_{klm} = \iint\limits_{D} x_{\mathcal{T}}^k(u,v) y_{\mathcal{T}}^l(u,v) z_{\mathcal{T}}^m(u,v) \sqrt{EG - F^2} \mathrm{d}u \mathrm{d}v \tag{3}$$

with the coefficients of the first fundamental form $E = x_u^2 + y_u^2 + z_u^2$, $G = x_v^2 + y_v^2 + z_v^2$ and $F = x_u x_v + y_u y_v + z_u z_v$, where $x_u = \frac{\partial x_{\mathcal{T}}(u,v)}{\partial u}$ and so forth. D is the domain of the parameters u and v. This was already pointed out in [24] and used to uniformly sample the triangles. We specify (3) for exact computation of the triangle area moments. By using the parameterization $P_{\mathcal{T}}(u,v)$, we reduce the computation of the surface moments of a triangle $\mathcal{T}$ to the computation of the area moments m'_{pq} of D, the definition domain of u, v. We will show these area moments m'_{pq} to be constant for all $P_{\mathcal{T}_i}(u,v)$. In a first step we find a proper parameterization for the general triangle $\mathcal{T}$. The second step shows that the computation of the surface moments of the parameterized triangle can be put down to the computation of the constant area moments of D.

First, we describe the triangle $\mathcal{T}$ as a simplex in $\mathbb{R}^3$,

$$\mathcal{T} = \alpha\mathbf{c}_1 + \beta\mathbf{c}_2 + \gamma\mathbf{c}_3 \text{ with } \alpha, \beta, \gamma \geq 0 \text{ and } \alpha + \beta + \gamma = 1. \tag{4}$$

It follows that $\gamma = 1 - \alpha - \beta$, and by rearranging we yield

$$\mathcal{T} = \alpha(\mathbf{c}_1 - \mathbf{c}_3) + \beta(\mathbf{c}_2 - \mathbf{c}_3) + \mathbf{c}_3 \text{ with } \alpha + \beta \leq 1. \tag{5}$$

By setting $u = \alpha$ and $v = \beta$ in (5) we reach the parameterization $P_{\mathcal{T}}(u,v) = (x_{\mathcal{T}}(u,v), y_{\mathcal{T}}(u,v), z_{\mathcal{T}}(u,v))^T$ of the triangle $\mathcal{T}$ with corners $\mathbf{c}_j = (x_j, y_j, z_j)^T$,

$$P_{\mathcal{T}}(u,v) = u\begin{pmatrix} x_1 - x_3 \\ y_1 - y_3 \\ z_1 - z_3 \end{pmatrix} + v\begin{pmatrix} x_2 - x_3 \\ y_2 - y_3 \\ z_2 - z_3 \end{pmatrix} + \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}. \tag{6}$$

From (5) we see that α and β and hence u and v do not depend on the triangle corners. Therefore, the domain is

$$D = \{(u,v) : u, v \geq 0, u + v \leq 1\} \tag{7}$$

for the parameterizations $P_{\mathcal{T}_i}(u,v)$ of each triangle $\mathcal{T}_i$. The area moments m'_{pq} of D are calculated by

$$m'_{pq} = \iint_D u^p v^q \mathrm{d}u\mathrm{d}v, \tag{8}$$

and the relevant area moments up to third order are: $m'_{00} = \frac{1}{2}$, $m'_{01} = m'_{10} = \frac{1}{6}$, $m'_{02} = m'_{20} = \frac{1}{12}$, $m'_{11} = \frac{1}{24}$, $m'_{03} = m'_{30} = \frac{1}{20}$ and $m'_{12} = m'_{21} = \frac{1}{60}$.

Second, we show that computing the moments m_{klm} can be reduced to the calculation of the moments m'_{pq} (9). We achieve the representation (9) by setting $C = \sqrt{EG - F^2}$, substituting (6) in to (3) and using (8).

By this means we compute the surface moments m^i_{klm} for all triangles $\mathcal{T}_i \in \mathcal{S}$ and finally establish the exact 3D surface moments M_{klm} of the surface triangulation $\mathcal{S}$. We compute exact surface moments M_{klm} without sampling.

Using the 3D surface moments M_{klm}, we finally compute the 11 3D moment invariants I^2_{22}, I^2_{222}, …, I^3_{1113} proposed by Lo and Don [11]. These invariants include moments up to third order. Due to space restrictions we are not able to present the definition of these invariants, but [11] gives an extensive presentation of how to derive and compute these 3D moment invariants. We denote them the same way to relieve identification.

3.2. Registration by Optimal Point Assignment

Having two surface triangulations $\mathcal{S}_1$, $\mathcal{S}_2$ as input data, we compute the 3D moment invariants I^2_{22}, …, I^3_{1113} for each vertex (3D triangle corner $\in \mathcal{S}_w$, $w \in \{1,2\}$) of each surface. Specifically, the origin is set into the respective vertex and moments are non-centralized. This yields *3D moment invariants as vertex descriptors* characterizing the position of the vertex within the considered surface or, in other words, characterizing the surface around the considered vertex. The descriptors are calculated from the surface itself without any fitting or sampling, and the property of invariance is derived algebraically.

We construct the cost matrix necessary for the Hungarian method using the Euclidean distances between vertex descriptors. The Hungarian method [10] assures an assignment of vertices with minimum overall cost and also works for point sets of unequal size. We thus register the surface triangulations $\mathcal{S}_1$, $\mathcal{S}_2$ by assigning vertex descriptors – and hence the vertices – of the respective surfaces to each other without necessarily assigning all descriptors.

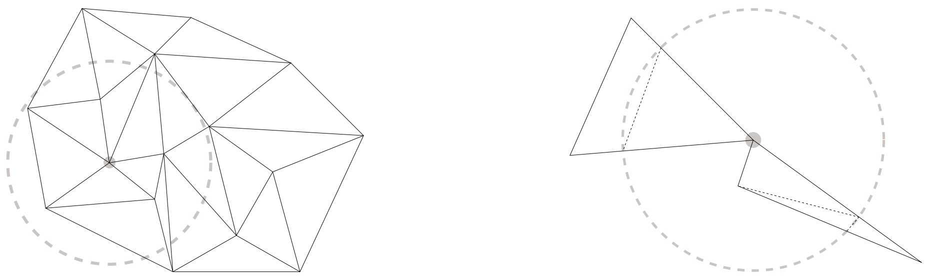
The assignment of surface points rises the discretization problem: We want to assign vertices of $\mathcal{S}_1$ to vertices of $\mathcal{S}_2$ that were sampled at the same position of the *real-world surface* $\mathfrak{S}$. Therefore, we have to assume structure-preserving sampling by $\mathcal{S}_1$, $\mathcal{S}_2$ and at least one $\mathcal{S}_j$ dense enough to roughly cover the sampling positions of the other surface triangulation. The practical meaning of this statement is that a human observer should be able to identify both surfaces referencing the same object, which is a natural condition for a solvable registration task.

3.3. Local Surface Moment Invariants and Selection of Descriptors

In Sect. 3.1 we presented a way to compute vertex descriptors from surface moment invariants. Naturally, these descriptors only characterize the part of the surface that is used to compute the surface moments. Using the whole surface $\mathcal{S}_1$ for the computation of a descriptor covers maximum information and, hence, produces features of maximum distinctiveness and robustness. On the other hand, this way is viable only if surface $\mathcal{S}_2$ is as complete as $\mathcal{S}_1$ and if the two surface triangulations represent exactly the same real-world surface $\mathfrak{S}$. In the presence of only partial overlap this assumption is violated. One way out is to calculate surface moments from an infinitesimal small surface region which results in a high sensitivity to noise.

We choose an intermediate way between global and infinitesimal small regions. For each vertex we use a local

$$
\begin{aligned}
&m_{klm} = \\
&= C \iint\limits_D [(u(x_1 - x_3) + v(x_2 - x_3) + x_3)^k (u(y_1 - y_3) + v(y_2 - y_3) + y_3)^l (u(z_1 - z_3) + v(z_2 - z_3) + z_3)^m] \mathrm{d}u\mathrm{d}v \\
&= C \iint\limits_D [u^{k+l+m}(x_1 - x_3)^k (y_1 - y_3)^l (z_1 - z_3)^m + \ldots + x_3^k y_3^l z_3^m] \mathrm{d}u\mathrm{d}v \\
&= C[(x_1 - x_3)^k (y_1 - y_3)^l (z_1 - z_3)^m \iint\limits_D u^{k+l+m} \mathrm{d}u\mathrm{d}v + \ldots + x_3^k y_3^l z_3^m \iint\limits_D \mathrm{d}u\mathrm{d}v] \\
&= C[(x_1 - x_3)^k (y_1 - y_3)^l (z_1 - z_3)^m m'_{(k+l+m)0} + \ldots + x_3^k y_3^l z_3^m m'_{00}].
\end{aligned} \tag{9}
$$



(a) Local surface region inside a sphere around vertex $\mathbf{V}$ (gray circular area).

(b) Approximation of triangles that have one or two points inside the sphere. One or two new edges (black dashed lines) and, thereby, new triangles are created.

Figure 1. Determination and approximation of a local surface region around vertex $\mathbf{V}$ of the surface. Vertex descriptor is computed with respect to this local region.

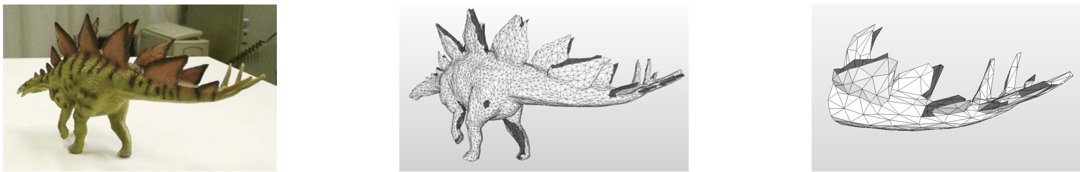
surface that is inside a sphere of radius r around the respective vertex. Figure 1 illustrates our approach. For the calculation of the moment invariants with respect to vertex $\mathbf{V}$ of the surface we compute the surface moments of the triangles inside the sphere around $\mathbf{V}$. If a triangle has one or two corner(s) outside the sphere, then we approximate the part inside the sphere by creating new triangles inside the original one using the intersection points of the triangle edges and the sphere, cf. Fig. 1. The *local* surface moments endow *local* surface moment invariants and, thus, a descriptor of the local surface around vertex $\mathbf{V}$. This means that the equality of these local descriptors claims equality of the surfaces only within each sphere around the respective vertices. By this means we handle partial overlap of the input surface triangulations.

The local surface restriction for the calculation of descriptors causes the need to filter out weak descriptors, as also stated in [19]. An obvious example of such a weak descriptor is a point within a planar region. This descriptor can be assigned to any other descriptor from a planar region without raising an assignment error. As a consequence, structural information within the local surface region is necessary to achieve distinctive point descriptors. This is the *aperture problem*. Our current solution to this problem is a statistical analysis of each input surface as described in the following. First, we calculate the maximum Euclidean distance between descriptors $d_{\max}$ of vertices on the surface. Second, we compute a local measure of distinctiveness for the considered vertex. For a vertex $\mathbf{V}$ and the local surface region inside the sphere with radius r we calculate the mean distance $\overline{d}_{\mathrm{loc}}$ between the descriptor of $\mathbf{V}$ and the descriptors of all other vertices inside the sphere. By comparing $\overline{d}_{\mathrm{loc}}$ to $d_{\max}$ and thresholding we decide if the point is used for assignment.

4. Experimental Evaluation

In the following we present quantitative evaluations of the method described in the previous sections. We show the benefits of our approach with respect to different point densities of the surface triangulations, various noise levels and overlap. In conclusion of the experimental results and stating the fact that our registration method does not require any initial solution, we like to emphasize its capabilities as a preprocessing to the ICP algorithm.

Figure 2 illustrates the test data used. We obtained a high quality 3D reconstruction of the dinosaur figure by using a fringe projection measurement system. All surface triangulations are estab-



(a) Real-world figure. Length about 200 mm.

(b) 3D reconstruction of the whole figure with 10^4 faces.

(c) Part (tail) of the full 3D reconstruction with 500 faces.

Figure 2. Test data. Dinosaur figure and 3D surface triangulations with partial overlap and different point densities.

Table 1. Evaluating registration of surface triangulations with different point densities. Our method proofs to be robust. Alignment convergence is reached for strongly differing densities.

V_1/F_1	V_2/F_2	runtime (s)	e_V (mm)	ICP convergence	e_{ICP} (mm)
5274/10000	5274/10000	105.14	0.00	yes	0.00
5274/10000	2670/5000	65.79	0.64	yes	0.10
5274/10000	558/1000	54.30	1.26	yes	0.20
5274/10000	290/500	52.57	8.97	yes	0.22
5274/10000	58/100	52.48	84.12	no	/

lished by standard Delaunay triangulation and a surface simplification algorithm [7] included in `Meshlab` (`http://meshlab.sourceforge.net/`). With this software, we also achieved 3D models of the same object with different point densities and different point distributions.

Throughout the experiments we use 3D moment invariants with respect to Euclidean transformations. For the quantitative evaluation of the alignment quality we determine the error e_V (mm) as the mean vertex-to-closest-vertex distance between the input surfaces. This measure describes the alignment accuracy of our coarse registration. After the registration step, we perform an ICP alignment using the `Matlab` implementation from [13]. By visual inspection we check for ICP convergence to the true solution and note the resulting alignment error e_{ICP} (mm), which is the mean vertex-to-surface distance. We assume the fact to be known that the ICP algorithm needs an initial solution inside its basin of convergence [16] to succeed. Thus, we omit tests on ICP alone and only give the ICP results using our coarse registration as the initial step. The noted runtimes (system: Core2 Duo, 2.4 GHz, 4 GB RAM) comprise all steps of the method, in particular calculating the surface moments, calculating the invariants and running the Hungarian method.

Evaluating the influence of different point densities. For the first experiment we reduce the density of the initial 3D model from Fig. 2(b) to yield surfaces $\mathcal{S}_j$ with V_j vertices and F_j faces (triangles) by using the surface simplification algorithm [7] implemented in `Meshlab 1.1.1`. Due to the complexity of the dinosaur, see Fig. 2, each reduction of the point density comes with a reduction of the preserved surface information. The registration results are listed in Table 1. We note the vertex/face numbers of each surface, which indicate the loss of information about the surface structure. Runtimes, error values and ICP convergence information are also given. In order to obtain results that are meaningful with respect to different point densities we compute features using the whole surface and, thus, eliminate effects arising from the size of a local region, which we will evaluate later on.

As seen from Table 1, our method produced suitable initial solutions for the ICP algorithm even with extremely different point densities (10000 $\leftrightarrow$ 500 faces). The method failed when applied to a pair of surfaces with 5274 vertices / 10000 faces and 58 vertices / 100 faces. The reason is the large loss in information when representing the complex dinosaur model with only 58 points. In case of such a strong simplification, whole object parts like the plates of the corselet vanish.

Evaluating the influence of additional noise. The robustness to noise is crucial for registration methods that are used with real data from practical measurements. Therefore, we evaluate our coarse registration method with respect to noisy input data. We register surfaces $\mathcal{S}_1$ (2670 vertices / 5000 faces) and $\mathcal{S}_2$ (558 vertices / 1000 faces), where each vertex coordinate of $\mathcal{S}_2$ is disturbed by Gaussian noise $\mathcal{N}(0, \sigma^2)$. This means that each of the $x/y/z$-coordinates of each vertex is independently disturbed by Gaussian noise with standard deviation σ. The registration results are listed in Table 2. All error values are mean values from ten independent cycles of disturbance and registration. The runtime was about 14 s for each cycle.

Table 2 shows that our registration method provided valuable coarse alignments with noise levels up to $\sigma = 2$ mm. That means the 3D model of the dinosaur of length

Table 2. Evaluating registration of surface triangulations (5000 $\leftrightarrow$ 1000 faces) with Gaussian noise of different levels. The coarse alignment still works with $\sigma = 2$ mm, i. e. 99.73% of all vertex $x/y/z$-coordinates are disturbed up to 6 mm in each dimension within the 3D model of length 200 mm.

σ (mm)	$\overline{e}_V$ (mm)	ICP convergence		$\overline{e}_{ICP}$ (mm)
		yes	no	
0.0	2.15	10x	0x	0.26
0.1	1.69	10x	0x	0.26
0.5	2.07	10x	0x	0.28
1.0	2.17	10x	0x	0.33
1.5	3.05	10x	0x	0.40
2.0	4.09	10x	0x	0.46
2.5	30.09	1x	9x	0.51

200 mm was disturbed in such a way that 99.73% of all vertex $x/y/z$-coordinates were displaced up to 6 mm in each dimension, and the following ICP still converges in presence of such severe surface deformations.

Evaluating partial overlap and the sphere radius for local feature computation The partial overlap criterion is evaluated using the 3D surfaces from Figs. 2(b) and 2(c). Surface $\mathcal{S}_1$ is the whole 3D model of the dinosaur with 5274 vertices / 10000 faces, $\mathcal{S}_2$ is the tail (length about 110 mm) with 290 vertices / 500 faces from a less dense version of the 3D model. The moment invariants of each vertex are calculated with respect to the local surface region around the vertex which lies inside a sphere of radius r around the vertex, cf. Fig. 1.

First, Table 3 shows that our registration method can handle real partial overlap effectively. There is, compared to the size of the 3D models, a large range of valid values for the sphere radius r to achieve a suitable initial solution for the ICP algorithm: valid values of r are between 15 mm and 55 mm. Second, we notice that ICP convergence was not reached for all values of r. Although the configuration of r seems to be non-critical and may be performed by selecting r about one third of the largest extension of the smaller surface, this remains an open point requiring future work.

5. Conclusion and Future Work

In this paper we presented a method for coarse registration of 3D surface triangulations based on 3D moment invariants. We extended an idea known from 2D affine point pattern matching for the application in 3D. We incorporated the usage of surface triangulations, established exact surface moments without sampling, used local surface regions for the calculation of moment descriptors and we proposed a method to filter out non-discriminative descriptors. The method does not use any initial solution and is thus a reasonable preprocessing step to the ICP algorithm.

The conducted experiments give evidence that our registration method can handle different point densities and distributions, partial overlap and that it is robust to noise.

Nonetheless, the sphere radius for local surface moments is still set manually. Future work is necessary and could, for instance, determine an optimal value of this radius within a preprocessing step. The problem of selecting a suitable radius in the presence of similarity transformations should also be treated in future research. An improvement of selecting local descriptors used for alignment can be reached by a stronger criterion for real 3D surface features. If the surface features are ambiguous, like on technical objects, an incorporation of information about global positioning has to be tested. Furthermore, one can reconsider the restriction of assigning vertices only.

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Table 3. Evaluating registration in the presence of partial overlap. ICP convergence is reached for a large range of values for radius r.

r (mm)	5	10	15	20	25	30	35	40	45	50	55	60	65
runtime (s)	4.56	7.74	10.81	17.04	23.78	30.52	39.15	48.64	60.62	72.41	84.32	96.65	110.95
e_V (mm)	4.53	5.34	1.54	1.33	1.67	2.51	2.28	3.06	1.58	2.35	2.22	7.53	6.47
ICP convergence	no	no	yes	yes	yes	yes	yes	yes	yes	yes	yes	no	no
e_{ICP} (mm)	/	/	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	/	/

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